

TESTING COSMOLOGICAL MODELS AGAINST THE ABUNDANCE OF DAMPED LYMAN-ALPHA ABSORBERS

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ABSTRACT

We calculate the number of damped Ly α absorbers expected in various popular cosmological models as a function of redshift and compare our predictions with observed abundances. The Press-Schechter formalism is used to obtain the distribution of halos with circular velocity in different cosmologies, and we calibrate the relation between circular velocity and absorption cross-section using detailed gas dynamical simulations of a “standard” cold dark matter (CDM) model. Because of this calibration, our approach makes more realistic assumptions about the absorption properties of collapsed objects than previous, analytic calculations of the damped Ly α abundance. CDM models with $\Omega_0 = 1$, $H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$, baryon density $\Omega_b = 0.05$, and scale-invariant primeval fluctuations reproduce the observed incidence and redshift evolution of damped Ly α absorption to within observational uncertainty, for both COBE normalization ($\sigma_8 = 1.2$) and a lower normalization ($\sigma_8 = 0.7$) that better matches the observed cluster abundance at $z = 0$. A tilted ($n = 0.8$, $\sigma_8 = 0.7$) CDM model tends to underproduce absorption, especially at $z = 4$. With COBE normalization, a CDM model with $\Omega_0 = 0.4$, $\Omega_\Lambda = 0.6$ gives an acceptable fit to the observed absorption; an open CDM model is marginally acceptable if $\Omega_0 \geq 0.4$ and strongly inconsistent with the $z = 4$ data if $\Omega_0 = 0.3$. Mixed dark matter models tend not to produce sufficient absorption, being roughly comparable to tilted CDM models if $\Omega_\nu = 0.2$ and failing drastically if $\Omega_\nu = 0.3$.

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1. Introduction

Damped Ly α (DLA) absorbers — hydrogen absorption systems with neutral column density $N_{\text{HI}} \geq 10^{20.3} \text{ cm}^{-2}$ — are important tracers of high density structure in the early universe. They are far more common than quasars or luminous radio galaxies, and various lines of evidence, both coincidences of numbers and properties of observed systems, indicate that DLA systems are the high-redshift progenitors of “normal” present-day galaxies (see, e.g., Wolfe et al. 1995; Djorgovski et al. 1996; Fontana et al. 1996; Prochaska & Wolfe 1997). In this *Letter*, we use a semi-analytic method, calibrated against numerical simulations, to test the ability of current theories of structure formation to reproduce the observed abundance of DLA absorbers at $2 \leq z \leq 4$.

The first attempts to employ DLA absorption as a test of cosmological scenarios used the Press-Schechter (1974) formalism to predict the abundance of dark matter halos and adopted simplifying, usually conservative assumptions about the amount of gas within these halos that would cool and become neutral (Kauffmann & Charlot 1994; Mo & Miralda-Escudé 1994; for recent analyses along similar lines see Liddle et al. 1996ab). Ma & Bertschinger (1994) and Klypin et al. (1995) proceeded in a similar fashion, but computed the halo mass function from dissipationless N-body simulations. Kauffmann (1996) presented a more sophisticated semi-analytic treatment of DLA systems and disk galaxy formation in the standard cold dark matter (CDM) model, incorporating more realistic assumptions about the cooling and settling of gas within halos and subsequent star formation.

A fundamentally different approach was taken by Katz et al. (1996; hereafter KWHM), who used numerical simulations incorporating dark matter and gas in the presence of a photoionizing background to show that damped Ly α systems originate naturally from initial conditions similar to those which are believed to have produced the observed large-scale structure. In the KWHM simulations, damped absorption results from concentrations of dense gas embedded in more extended and more massive dark matter halos. The masses and abundances of these objects are comparable to those of galaxies, supporting the view that damped Ly α systems are a natural by-product of galaxy formation. Lower column density, Ly α forest absorption arises in more diffuse, highly photoionized structures, and

the combined results of KWHM and Hernquist et al. (1996) show that a simulation of the standard CDM model reproduces the observed distribution of neutral hydrogen column densities quite well over most of the range $10^{14} \text{ cm}^{-2} \leq N_{\text{HI}} \leq 10^{22} \text{ cm}^{-2}$.

Gardner et al. (1997; hereafter GKH) improved on KWHM by using a semi-analytic method to correct for damped absorption in halos below the KWHM resolution limit. Using KWHM’s simulation and higher resolution simulations focused on individual, collapsing halos, GKH compute the mean projected area $\alpha(v_c, z)$ over which a halo of circular velocity v_c produces damped Ly α absorption at redshift z . They convolve this function with the Press-Schechter formula for the abundance of halos as a function of v_c to obtain the total incidence of damped absorption in all halos. The correction for unresolved objects raises the KWHM predictions by roughly a factor of two, bringing them into good agreement with the observed abundance of damped absorbers, though the predicted number of Lyman limit systems remains low by about a factor of three. GKH also show that star formation (with the algorithm of Katz, Weinberg, & Hernquist 1996) has little effect on the predicted number of damped systems.

The method adopted in KWHM and GKH can be applied to other cosmological scenarios, and we believe that numerical simulations with gas dynamics will eventually allow the most robust confrontations between predicted and observed properties of DLA absorbers. However, the fully numerical approach is computationally very expensive, and it will be some time before it can be applied to a broad parameter space of cosmological models. This seems like an opportune moment to revisit the semi-analytic approach for several reasons. First, the theoretical ground has shifted somewhat since the first investigations, prompted in part by improved analyses of the COBE data (and the 4-year COBE data themselves) and in part by further studies of the theoretical models, especially the emergence of open-bubble inflation models as a viable and predictive setting for structure formation (e.g., Ratra & Peebles 1994; Bucher, Goldhaber, & Turok 1995; Yamamoto, Sasaki, & Tanaka 1995). Second, new data have improved the observational constraints, especially at $z \approx 4$ (Wolfe et al. 1995; Storrie-Lombardi et al. 1996). Third, and most significant from the point of view of this paper, the GKH results provide a way to calibrate the most uncertain element of the semi-analytic method, the relation between halo circular velocity and the cross-section for damped absorption. It seems plausible that this relation is driven primarily by “local” physics within the halo, and for this paper we will therefore assume that the function $\alpha(v_c, z)$ found by GKH for the standard CDM model also applies to other cosmological scenarios. With this assumption — uncertain, but probably the best approximation available until alternative models are investigated numerically — we can use the GKH method to compute the abundance of DLA absorbers in any specified cosmology with Gaussian initial conditions. We describe this method in

more detail in §2, then present our results and discuss their implications in §3.

2. Method

KWHM used TreeSPH (Hernquist & Katz 1989) to evolve to redshift $z = 2$ a periodic cube 22.22 comoving Mpc on a side containing dark matter and gas. The initial conditions were drawn from a CDM power spectrum with $\sigma_8 = 0.7$, $\Omega_0 = 1$, $h = 0.5^1$, and $\Omega_b = 0.05$, where σ_8 is the present-day rms mass fluctuation within a sphere of radius $8 h^{-1}$ Mpc, and Ω_b and Ω_0 are the present-day fractions of closure density in the form of baryons and matter respectively. The simulation included a spatially uniform background radiation field of intensity $J(\nu) = J_0(\nu_0/\nu)F(z)$, where ν_0 is the Lyman-limit frequency, $J_0 = 10^{-22}$ erg $\text{s}^{-1} \text{cm}^{-2} \text{sr}^{-1} \text{Hz}^{-1}$, and $F(z) = 0$ if $z > 6$, $F(z) = 4/(1+z)$ if $3 \leq z \leq 6$, and $F(z) = 1$ if $2 < z < 3$. More detailed descriptions of the simulation and methods can be found in GKHWH, KWHM, and Katz et al. (1996).

Given their limited particle number, KWHM were able to resolve halos only down to a mass corresponding to a virial circular velocity of $v_c \approx 100 \text{ km s}^{-1}$ (about $10^{11} M_\odot$ at $z = 2$; see, also Weinberg, Hernquist, & Katz 1997). Since Quinn, Katz, & Efstathiou (1996) find that halos with $v_c > 37 \text{ km s}^{-1}$ are capable of hosting damped Ly α absorbers, it is clear that KWHM’s simulation systematically underestimates the true prediction of this CDM model for the incidence of damped absorption. GKHWH combine the background radiation field of KWHM with the initial conditions of Quinn et al. (1996) to simulate small regions with sufficient detail to resolve halos with circular velocities below 50 km s^{-1} . The high resolution volumes are too small to calculate the number of damped systems (they were chosen to lie in regions where small halos would form; see Quinn et al. 1996), but by using the halos from both the high resolution and KWHM simulations, GKHWH estimate $\alpha(v_c, z)$, the cross-section for a halo of circular velocity v_c to produce damped Ly α absorption at redshift z . GKHWH show that $\alpha(v_c, z)$ can be fitted by a power law in v_c , with an exponential cutoff to reproduce the absence of absorbers in halos with $v_c < 37 \text{ km s}^{-1}$ (Quinn et al. 1996). The fitting procedure is described in GKHWH, which also lists parameters of the fitted relation at $z = 2, 3$, and 4 .

According to Press-Schechter theory, the number density $N(M, z)$ of collapsed objects

¹Throughout, $h \equiv H_0/(100 \text{ km s}^{-1} \text{Mpc}^{-1})$, where H_0 is Hubble’s constant today.

in the mass range $M \longrightarrow M + dM$ at redshift z is

$$N(M, z)dM = \sqrt{\frac{2}{\pi}} \frac{\rho_0}{M} \frac{\delta_c}{\sigma_0} \left(\frac{\gamma R_f}{R_*} \right)^2 \exp \left(\frac{-\delta_c^2}{2\sigma_0^2} \right) dM, \quad (1)$$

where ρ_0 is the mean comoving mass density and R_f is the Gaussian filter radius given by $M = (2\pi)^{3/2} \rho_0 R_f^3$. The parameters σ_0 , γ and R_* are related to moments of the power spectrum and are defined in Bardeen et al. (1986). As detailed in GKH, multiplying $N(M, z)$ by $\alpha(v_c, z)$ and integrating from v_c to infinity yields the number of damped absorbers per unit redshift residing in halos of circular velocity greater than v_c :

$$n(z, v_c) = \frac{dr}{dz} \int_{M(v_c)}^{\infty} N(M', z) \alpha(v'_c, z) dM', \quad (2)$$

where v'_c is the circular velocity at the virial radius of a halo of virial mass M' , and r is comoving distance. With $v_c = 37 \text{ km s}^{-1}$, the lower limit for damped absorption found by Quinn et al. (1996), equation (2) yields the total incidence of DLA absorption $n(z)$, defined to be the number of absorption systems per unit redshift with $N_{\text{HI}} \geq 10^{20.3} \text{ cm}^{-2}$. In this paper, we adopt the $\alpha(v_c, z)$ relations found by GKH for the standard CDM model but combine them with the halo abundance $N(M, z)$ predicted for other cosmological scenarios.

For all the Press-Schechter predictions we use a Gaussian filter with $\delta_c = 1.69$. This δ_c is higher than that conventionally used with a Gaussian filter because of the way that we identify halos and assign them circular velocities (see GKH), which we believe most closely corresponds to the conception of halos in the Press-Schechter formalism. As explained in GKH, this value of δ_c fits the numerically derived halo mass function of KWHM quite well. Our full procedure correctly predicts $n(z, v_c)$ in the KWHM simulations for damped systems in halos above the $v_c = 100 \text{ km s}^{-1}$ resolution limit. In our calculations of $N(M, z)$, we use the transfer functions of Bardeen et al. (1986) for cold dark matter models and Ma (1996) for mixed dark matter models.

3. Results

Table 1 compares the predicted numbers of DLA absorbers to the observational results reported by Storrie-Lombardi et al. (1996). These results are illustrated in Figure 1, which also includes the observational data of Wolfe et al. (1995). To aid the interpretation of this Figure, we show in Figure 2 the comoving number density of halos with $v_c > 37 \text{ km s}^{-1}$ obtained by integrating equation (1) for each model. Starting at high redshift, the halo abundance first rises as rms fluctuations on the $v_c \sim 37 \text{ km s}^{-1}$ mass scale go nonlinear, then declines as the first generation of halos above this threshold merges into a smaller

number of more massive halos. For some of our models, this decline has already begun by $z = 5$.

The model producing the most DLA absorption and the best agreement with the $z = 4$ data is CCDM: COBE-normalized, $\Omega_0 = 1$ CDM with scale-invariant ($n = 1$) primeval fluctuations and $h = 0.5$. However, this model is known to produce excessively massive clusters at $z = 0$. SCDM, the “standard” CDM model used in GKH, has an identical power spectrum shape but is normalized to produce more reasonable (still somewhat high) cluster masses, while disagreeing with the COBE results. In this case, $n(z)$ is lower due to the decrease in power (*cf.* Figure 2), but the model still matches the DLA observations within current uncertainties. It is possible to construct an $\Omega_0 = 1$ CDM model that simultaneously fits cluster masses and COBE data by further adjustments, such as tilting the primeval power spectrum to $n < 1$ and raising the baryon fraction. White et al. (1996) give an example of such a model, which we designate TCDM: with $n = 0.8$ and $\Omega_b = 0.1$, COBE normalization implies $\sigma_8 = 0.7$ just as for our SCDM model. However, the tilt and high Ω_b reduce the amount of small-scale power relative to SCDM, reducing the halo abundance at $z > 2$ (Figure 2) and producing an $n(z)$ significantly below the Storrie-Lombardi et al. (1996) data for $z = 3$ and $z = 4$. There are other combinations of n , Ω_b , and h that can match COBE and still have CDM, $\Omega_0 = 1$, and $\sigma_8 \sim 0.7$, but since the necessary changes all reduce the amount of power on small scales, the problem shown here is likely to be generic to all such models.

It is important to note that TCDM is the only model we analyze that has $\Omega_b h^2$ different from 0.0125, and we have not allowed the increased baryon density to change the DLA cross-section in our treatment here. It is not difficult to imagine that the increase in baryon density could alter the absorption produced in high-redshift halos. Larger Ω_b implies an increased gas supply for a halo of a given mass, which could simply lead to a higher incidence of damped absorption. On the other hand, it could also lead to more rapid cooling and more dense concentrations of cold collapsed gas, thereby decreasing $n(z)$. Consequently, our results for this model are not robust enough to rule it out until the influence of Ω_b on $n(z)$ has been more thoroughly investigated.

CDM models with a cosmological constant retain the flat universe preferred by inflationary models while decreasing the density of matter, permitting more reasonable cluster masses with a COBE normalization. They also match the shape of the power spectrum on galaxy cluster scales more closely than SCDM, and they avoid the severe problems of $\Omega_0 = 1$ models concerning the age of the universe and the cluster baryon fraction. Our LCDM model, COBE-normalized with $\Omega_0 = 0.4$ and $h = 0.65$, generally falls within the 1σ errors of the Storrie-Lombardi data except at $z = 4$. Many of the systems

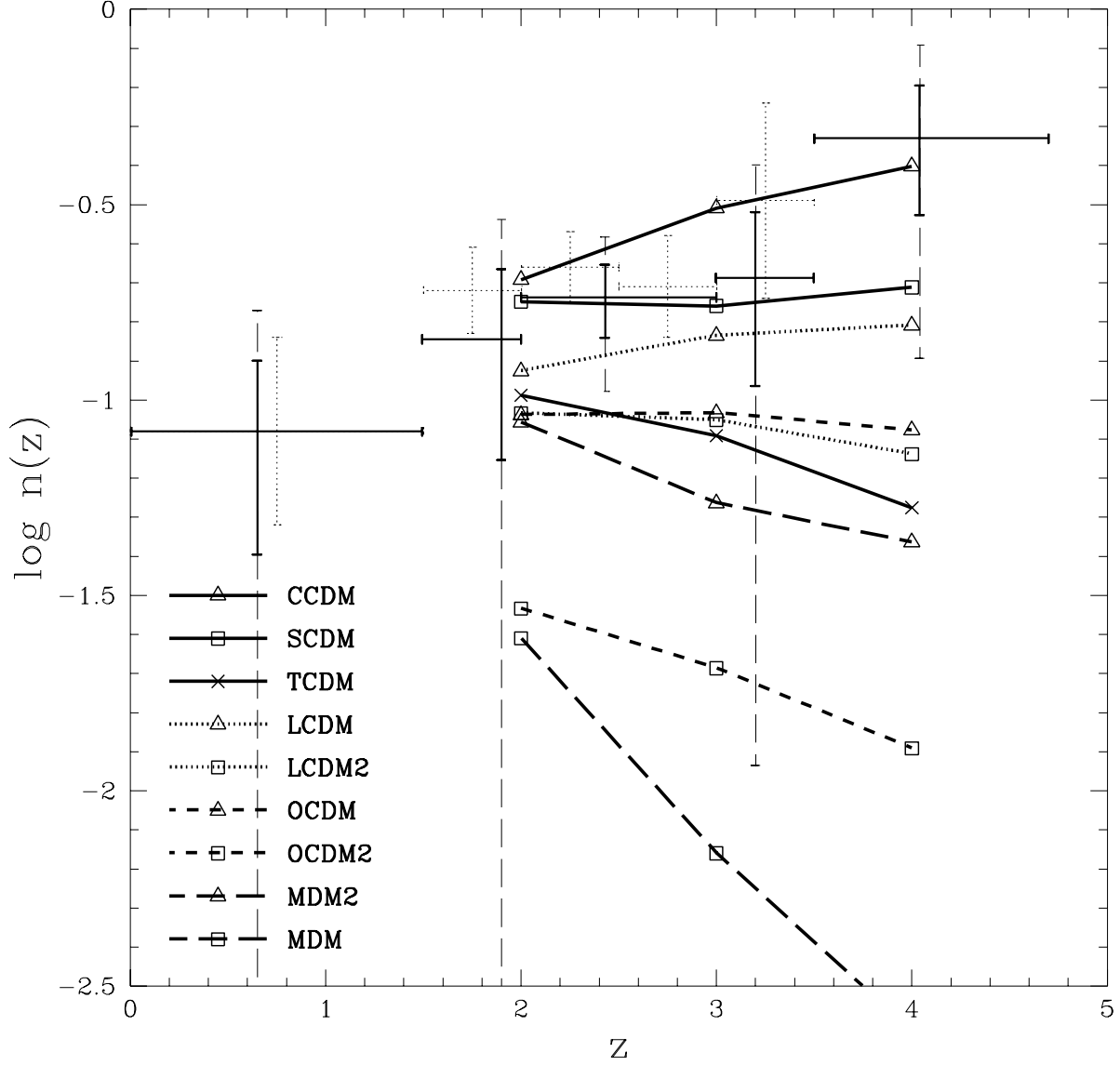


Fig. 1.— The predicted number $n(z)$ of DLA systems per unit redshift for various cosmological models vs. observational data. The solid error crosses (1σ) are reproduced from Storrie-Lombardi et al. (1996), with the vertical dashed error bars indicating 2σ error. The dotted error crosses (1σ) are taken from Wolfe et al. (1995). The heavy curves connecting $z = 2, 3, 4$ are values calculated for each model. The model names correspond to the names in Table 1, where their parameters are listed.

Name	σ_8	Ω_0	Ω_Λ	h	Other	$n(z)$		
						$z = 2$	$z = 3$	$z = 4$
CCDM ^a	1.2	1	0	0.5		0.203	0.310	0.397
SCDM	0.7	1	0	0.5		0.179	0.174	0.194
TCDM ^b	0.7	1	0	0.5	$n = 0.8$	0.103	0.0811	0.0530
LCDM ^a	1.1	0.4	0.6	0.65		0.119	0.146	0.155
LCDM2 ^c	0.79	0.4	0.6	0.65		0.0926	0.0889	0.0730
OCDM ^d	0.65	0.4	0	0.65		0.0916	0.0926	0.0836
OCDM2 ^d	0.45	0.3	0	0.65		0.0293	0.0206	0.0128
MDM ^e	0.782	1	0	0.5	$\Omega_\nu = 0.3$	0.0246	0.00692	0.00243
MDM2 ^e	0.808	1	0	0.5	$\Omega_\nu = 0.2$	0.0875	0.0545	0.0432
Observed ^f						$1.5 < z < 2.0$	$2.0 < z < 3.0$	$3.0 < z < 3.5$
						0.14 ± 0.073	0.18 ± 0.039	0.21 ± 0.097
						$3.5 < z < 4.7$		
							0.47 ± 0.17	

Table 1: The predicted abundance of DLA systems, $n(z)$, for a variety of cosmological models. $\Omega_\Lambda \equiv \Lambda/(3H_0^2)$ where Λ is the cosmological constant. References for model normalizations are listed below. The final row shows observed data and 1σ errors.

^aBunn & White (1996)

^bWhite et al. (1996)

^cCen et al. (1994), Kofman, Gnedin, & Bahcall (1993)

^dGorksi et al. (1996)

^eMa (1996)

^fStorrie-Lombardi et al. (1996)

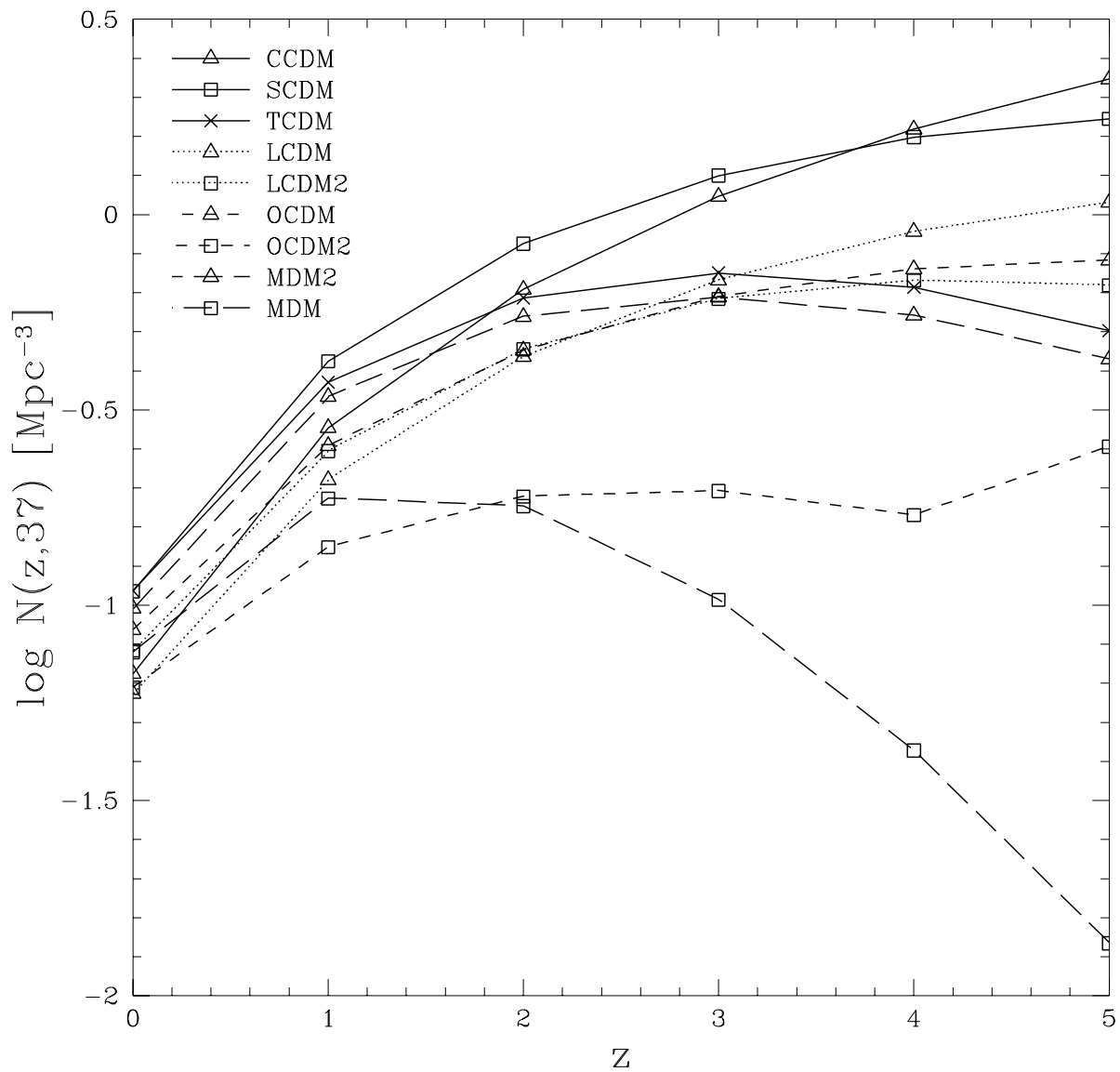


Fig. 2.— Comoving number density of halos with $v_c > 37$ km s⁻¹, $N(z, 37)$, vs. redshift. The models and labeling are as in Figure 1.

which contribute to the $z = 4$ data point have not yet been confirmed by high-resolution spectroscopy, however, so the value could decrease with future observations. Hence LCDM could be considered acceptable. LCDM2 is an identical model with the lower normalization favored by Cen et al. (1994). The lessened power of this model leads to a lower $n(z)$. Actually achieving this low normalization while being consistent with COBE would require a tilt or a lower h , which in turn would reduce small scale power, further exacerbating the problem.

It is also possible to preserve an inflationary scenario in an open CDM universe. OCDM shows a COBE-normalized open model similar to LCDM but with no cosmological constant. The open model has a lower mass fluctuation amplitude and hence a lower $n(z)$ than LCDM. An open CDM model with $\Omega_0 = 0.5$, $\sigma_8 \sim 0.9$ (not shown) skirts the lower end of the observational data and might be acceptable within the current uncertainties. The amount of small scale power implied by COBE normalization in open models decreases rapidly with decreasing Ω_0 (*cf.* Figure 2). For $\Omega_0 = 0.3$ (OCDM2), σ_8 is reduced to near 0.5, and the predicted $n(z)$ is well below the observations.

The most popular alternative to lowering Ω_0 is to add hot dark matter. This reduces σ_8 given by the COBE normalization and makes the shape of the power spectrum closer to that observed for galaxies. The first incarnations of this mixed dark matter model had $\Omega_\nu = 0.3$, which we reproduce here as MDM using the power spectrum and normalization given in Ma (1996). This model has strongly suppressed structure formation at high redshift (*cf.* Figure 2), and it drastically underproduces DLA absorption. Largely in recognition of this problem, $\Omega_\nu = 0.2$, shown here as MDM2, has become the favored version. While more palatable than MDM, this model still falls 2.8σ short for $2 < z < 3$ and 2.5σ below observations at $z = 4$. This problem could possibly be alleviated by further modifications such as “anti-tilting” the primeval spectrum to $n > 1$ or raising the Hubble constant to $h > 0.5$, but these changes would raise σ_8 , which is already too high to yield a good match to cluster abundances for $\Omega_0 = 1$ (White, Efstathiou, & Frenk 1993; Eke, Cole, & Frenk 1996). The most optimistic reading would be that mixed-dark matter can barely squeak by the combined constraints of cluster abundances on one hand and DLA’s on the other. Improvements in the observational data of DLA’s and the firming up of theoretical predictions by targeted simulations will probably close this window of parameter space.

These results are consistent with the findings of Mo & Miralda-Escudé (1994), in which LCDM underproduces absorption at $2 \lesssim z \lesssim 3$ by roughly a factor of three and MDM (with $\Omega_\nu = 0.3$) fails in this range by many orders of magnitude. Kauffmann & Charlot (1994) also conclude that standard CDM gives the best match to observations, remarking upon the inability of MDM models to produce the necessary cold, collapsed

gas at higher redshifts. Ma & Bertschinger (1994) find that an $\Omega_\nu = 0.2$ MDM model achieves rough agreement with the observations in the range $2.5 < z < 3.5$, but they assume that the distribution of gas matches the distribution of dark matter in the halos of their (dissipationless) simulation. The KWHM simulations show that only a fraction of the gas in a virialized halo cools and becomes neutral and that it is distributed primarily in one or more small, dense knots; both effects can reduce the damped Ly α absorption cross-section of a given halo. Ma et al. (1997) have undertaken a new study of DLA absorption in the mixed dark matter model, calibrating the Ma & Bertschinger (1994) dissipationless method against the KWHM simulation of SCDM, and they also find that the $\Omega_\nu = 0.2$ model underproduces the observed DLA absorption.

The global cosmological parameters in the CCDM and MDM models are the same as those in the SCDM model simulated by KWHM, and we are therefore fairly confident that our method can accurately calibrate their predictions of $n(z)$. The TCDM, LCDM, and OCDM models, however, adopt different values of Ω_b , Ω_0 , h , and Λ , altering the relations between overdensity and gas cooling rate and between redshift and the age of the universe. Our assumption that $\alpha(v_c, z)$ is close to that of the SCDM model is therefore more suspect. GKHW find that the DLA cross-section at fixed v_c decreases with time as the gas condenses into tighter configurations. If we assumed a similar trend across changes of cosmological parameters, then our predictions of $n(z)$ for TCDM, LCDM, and OCDM would go down, so in this sense our conclusions about these models are likely to be conservative. However, more detailed numerical studies will be required before we can build a definitive case against them.

As a concluding comment, we can generalize a point that we have already made regarding the mixed dark matter models. The model that produces the best match to the high-redshift DLA data, CCDM, is grossly inconsistent with the mass function of galaxy clusters today. The SCDM model, which fits the DLA data acceptably, is at best barely compatible with the cluster data, which imply $\sigma_8 \approx 0.55$ for $\Omega_0 = 1$ rather than $\sigma_8 = 0.7$ (White et al. 1993; Eke et al. 1996). Our LCDM model skirts the low end of the DLA data and the high end of the cluster data (see Cole et al. 1997). Clearly these are specific instances of a general problem, within the broad class of Gaussian/inflationary models considered here, since most parameter changes that reduce cluster masses also reduce DLA abundances. Between the Scylla of the damped Ly α systems and the Charybdis of the cluster mass function lies a narrow channel that few cosmological models will pass.

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